Neutrino Oscillation Scenarios and GUT Model Predictions[1]

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Abstract

The present experimental situation regarding neutrino oscillations is first summarized, followed by an overview of selected grand unified models which have been proposed to explain the various scenarios with three active neutrinos and their right-handed counterparts. Special attention is given to the general features of the models and their ability to favor some scenarios over others.

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1 Three-Active-Neutrino Oscillation Scenarios

1.1 Atmospheric Neutrinos

Recent results from the Super-Kamiokande Collaboration [1] involving atmospheric neutrinos convincingly favor muon-neutrinos oscillating into tau-neutrinos rather than into light sterile neutrinos. The latter possibility is ruled out at the 99% confidence level. In terms of the oscillation parameters, $\Delta m_{ij}^2 \equiv m_i^2 - m_j^2$ and $\sin^2 2\theta_{atm}$, the best fit values obtained are

$$
\begin{aligned}
\Delta m_{32}^2 &= 3.2 \times 10^{-3} \text{ eV}^2, \\
\sin^2 2\theta_{23} &= 1.000,
\end{aligned}
\tag{1}
$$

with the latter related to the neutrino mixing matrix elements by $\sin^2 2\theta_{atm} = 4|U_{\mu 3}|^2|U_{\tau 3}|^2$.

1.2 Solar Neutrinos

The situation regarding solar neutrinos is considerably less certain. The recent analysis [2] by the Super-Kamiokande Collaboration involving their 1117 day sample together with the data from the Chlorine [3] and Gallium [4] experiments favor the large mixing angle MSW [5] solution (LMA) and possibly the LOW solution over the small mixing angle (SMA) and vacuum (VAC) solutions, with the latter two being ruled out at the 95% confidence level. Several theory groups analyzing the same data suggest instead that while the LMA solution is favored, the other solutions are still viable at the 95% c.l. In fact, a continuum solution – the quasi-vacuum solution (QVO) – stretches between the LOW and VAC regions with $\tan^2 \theta_{sol} \gtrsim 1.0$. The best fit points in the various parameter regions found in a recent analysis by Gonzalez-Garcia and Peña-Garay [6] are given by

$$
\begin{aligned}
SMA: \quad & \Delta m_{21}^2 = 5.0 \times 10^{-6} \text{ eV}^2, \\
& \sin^2 2\theta_{12} = 0.0024, \\
& \tan^2 \theta_{12} = 0.0006, \\
LMA: \quad & \Delta m_{21}^2 = 3.2 \times 10^{-5} \text{ eV}^2, \\
& \sin^2 2\theta_{12} = 0.75, \\
& \tan^2 \theta_{12} = 0.33,
\end{aligned}
$$

$$LOW: \quad \Delta m_{21}^2 = 1.0 \times 10^{-7} \text{ eV}^2,$$
$$\sin^2 2\theta_{12} = 0.96,$$
$$\tan^2 \theta_{12} = 0.67,$$
$$QVO: \quad \Delta m_{21}^2 = 8.6 \times 10^{-10} \text{ eV}^2,$$
$$\sin^2 2\theta_{12} = 0.96,$$
$$\tan^2 \theta_{12} = 1.5, \tag{2}$$

Note that θ_{12} is in the second octant or "dark side" for the quasi-vacuum region. An even more recent analysis, [7] which includes the CHOOZ reactor constraint, [8] modifies the above numbers slightly, and sets $\tan^2 \theta_{13} = 0.005$.

1.3   Maximal and Bimaximal Mixings

The Maki-Nakagawa-Sakata (MNS) neutrino mixing matrix, analogous to the CKM mixing matrix, can be written as

$$U_{MNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \tag{3}$$

in terms of $c_{12} = \cos\theta_{12}$, $s_{12} = \sin\theta_{12}$, etc. With the oscillation parameters relevant to the scenarios indicated above, we can approximate $\theta_{13} = 0^o$ and $\theta_{23} = 45^o$ whereby Eq. (3) becomes essentially

$$U_{MNS} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12}/\sqrt{2} & c_{12}/\sqrt{2} & 1/\sqrt{2} \\ s_{12}/\sqrt{2} & -c_{12}/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}, \tag{4}$$

where the light neutrino mass eigenstates are given in terms of the flavor states by

$$\nu_3 = \tfrac{1}{\sqrt{2}}(\nu_\mu + \nu_\tau),$$
$$\nu_2 = \nu_e \sin\theta_{12} + \tfrac{1}{\sqrt{2}}(\nu_\mu - \nu_\tau)\cos\theta_{12}, \tag{5}$$
$$\nu_1 = \nu_e \cos\theta_{12} - \tfrac{1}{\sqrt{2}}(\nu_\mu - \nu_\tau)\sin\theta_{12},$$

For the SMA solution, $\theta_{12} = 1.4^o$, while the three large mixing solar solutions differ from maximal in that the angle is approximately 30° for the LMA, 39° for the LOW, and 51° for the QVO solutions. In contrast, the CKM quark mixing matrix is approximately

$$V_{CKM} = \begin{pmatrix} 0.975 & 0.220 & 0.0032e^{-i\delta} \\ -0.220 & 0.974 & 0.040 \\ 0.0088 & -0.040 & 0.999 \end{pmatrix}. \tag{6}$$

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An important issue to be answered is why $U_{\mu 3} \simeq 1/\sqrt{2}$ is so much larger than $V_{cb} \simeq 0.040$.

Maximal mixing of two neutrino mass eigenstates can arise if the two states are nearly degenerate in mass, i.e., the neutrinos are pseudo-Dirac. It can also arise if the determinant of the 2×2 submatrix nearly vanishes. For example,

$$\begin{pmatrix} x^2 & x \\ x & 1 \end{pmatrix} \rightarrow \lambda = 0, \ 1 + x^2, \ \psi_0 \sim \begin{pmatrix} 1 \\ -x \end{pmatrix}, \ \psi_+ \sim \begin{pmatrix} x \\ 1 \end{pmatrix}, \tag{7}$$

and the components are comparable for $x \simeq 1$. The first situation is relevant for the QVO and LOW near maximal mixings, while the second is more relevant for the atmospheric and LMA mixings where a sizable hierarchy is expected to be present.

Finally, it should be noted that the U_{MNS} mixing matrix is the product of two unitary transformations diagonalizing the charged lepton mass matrix L and the light neutrino mass matrix M_ν:

$$U_{MNS} = U_L^\dagger U_\nu, \tag{8}$$

where by the seesaw mechanism $M_\nu = -N^T M_R^{-1} N$ is given in terms of the Dirac neutrino matrix N and the right-handed Majorana matrix M_R. The structure of U_{MNS} is then determined by the three matrices N, M_R and L, one of which or in concert can play a role in the maximal or bimaximal mixing pattern.

2 Types of Neutrino Models and Possible Unifications

Neutrino models can be characterized as belonging to one of three types for the purpose of this talk.

- Those involving only left-handed fields leading to a left-handed Majorana mass matrix with no Dirac neutrino mass matrix present. The Zee model [9] is a prime example. Typically lepton number is violated by two units, or an $L = -2$ isovector Higgs field is introduced. A newly-defined lepton number $\bar{L} \equiv L_e - L_\mu - L_\tau$ is often taken to be conserved. The ultralight neutrino masses, however, are not easily understood.

- Models in which both left-handed and right-handed fields are present. With no Higgs contributions to the left-handed Majorana mass matrix, the seesaw mechanism readily yields ultralight neutrino masses, provided the right-handed Majorana masses are in the range of $10^5 - 10^{14}$ GeV. Such masses are naturally obtained in SUSY GUT models with $\Lambda_G = 2 \times 10^{16}$ GeV.

- Models in which neutrinos probe higher dimensions. Right-handed neutrinos which are singlets under all gauge symmetries can enter the bulk. With large extra dimensions and the compactification scale much lower than the string scale, a modified seesaw mechanism can generate ultralight neutrino masses.

I shall restrict my attention in this overview to models involving both left-handed and right-handed neutrinos. In this workshop, Tobe [10] addresses purely left-handed neutrino models in the context of R-parity violation, while Mohapatra [11] considers models involving higher dimensions.

Both the nonsupersymmetric standard model (SM) and the minimum supersymmetric extended version (MSSM) involve no right-handed neutrinos and just one or two Higgs doublets, respectively. Hence no renormalizable mass terms can be constructed for the neutrinos; moreover, the renormalizable mass terms which are present for the quarks and charged leptons have completely arbitrary Yukawa couplings. In order to reduce the number of free parameters and thereby achieve some detailed predictions for the mass spectra of the fundamental particles, some flavor and/or family unification must be introduced. This is generally done in the context of supersymmetry for which the desirable feature of gauge coupling unification obtains.

Flavor or vertical symmetry has generally been achieved in the framework of Grand Unified Theories (GUTs) which provide unified treatments of quarks and leptons, as (some) quarks and leptons are placed in the same multiplets. Examples involve $SU(5)$, $SU(5) \times U(1)$, $SO(10)$, E_6, etc.

The introduction of a family or horizontal symmetry, on the other hand, enables one to build in an apparent hierarchy for masses of comparable flavors belonging to different families. Such a symmetry may be discrete as in the case of Z_2, S_3, $Z_2 \times Z_2$, etc. which results in multiplicative quantum numbers. A continuous symmetry such as $U(1)$, $U(2)$, $SU(3)$, etc., on the other hand, results in additive quantum numbers and may be global or local (and possibly anomalous).

Combined flavor and family symmetries will typically reduce the number of model parameters even more effectively. On the other hand, the unification of flavor and family symmetries into one single group such as $SO(18)$ or $SU(8)$, for example, has generally not been successful, as too many extra states are present which must be made superheavy.

3 Froggatt-Nielsen-type Models with Anomalous $U(1)$ Family Symmetry

In 1979 Froggatt and Nielsen [12] added to the SM a scalar singlet "flavon" ϕ_f, which gets a VEV, together with heavy fermions, $(F, \bar{F})$, in vector-like representations, all of which carry $U(1)$ family charges. With $U(1)$ broken at a scale M_G by $\langle \phi \rangle / M_G \equiv \lambda \sim (0.01 - 0.2)$, the light and heavy fermions are mixed; hence λ can serve as an expansion parameter for the quark and lepton mass matrix entries. No GUT is involved, although M_G is some high unspecified scale.

This idea received a revival in the past decade when it was observed by Ibanez [13] that string theories with anomalous $U(1)$'s generate Fayet-Iliopoulos D-terms which trigger the breaking of the $U(1)$ at a scale of $O(\lambda)$ below the cutoff, again providing a suitable expansion parameter. The λ^n structure of the mass matrices can be determined from the corresponding Wolfenstein λ structure of the CKM matrix and the quark and lepton mass ratios, where different $U(1)$ charges are assigned to each quark and lepton field.

By careful assignment of the $U(1)$ charges, Ramond and many other authors [3] have shown that maximal mixing of $\nu_\mu \leftrightarrow \nu_\tau$ can be obtained, while the SMA solution for $\nu_e \leftrightarrow \nu_\mu$, ν_τ is strongly favored. Since then, other authors [15] have applied the technique in the presence of $SU(5)$ or $SO(10)$ to get also the QVO or LOW solutions. Very recently, Kitano and Mimura [16] have considered $SU(5)$ and $SO(10)$ models in this framework with an $SU(3) \times U(1)$ horizontal symmetry to show that the LMA solution can also be obtained. But with these types of models, the coefficients (prefactors) of the λ powers can not be accurately predicted.

4 Predictive SUSY GUT Models

With the minimal $SU(5)$ SUSY GUT model extended to include the left-handed conjugate neutrinos, the matter fields are placed in $\bar{\mathbf{5}}$ and $\mathbf{10}$ representations according to

$$\bar{5}_i \supset (d_\alpha^c, \ \ell, \ \nu_\ell)_i, \quad 10_i \supset (u_\alpha, \ d_\alpha, \ u_\alpha^c, \ \ell^c)_i, \quad 1_i \supset (\nu^c)_i, \quad \alpha = 1, 2, 3. \tag{9}$$

while the Higgs fields are placed in the adjoint and fundamental representations

$$\Sigma(24), \ H_u(5), \ H_d(\bar{5}). \tag{10}$$

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[3]For reference listings in two more comprehensive reviews, see [14].

The $SU(5)$ symmetry is broken down to the MSSM at a scale Λ_G with $\langle\Sigma\rangle$ pointing in the $B-L$ direction, but doublet-triplet splitting must be done by hand. The electroweak breaking occurs when the H_u and H_d VEV's are generated.

The number of Yukawa couplings has now been reduced in the Yukawa superpotential, and the fermion mass matrices exhibit the symmetries, $M_U = M_U^T$, $M_D = M_L^T$. This implies $m_b = m_\tau$ at the GUT scale, but also $m_d/m_s = m_e/m_\mu$ which is too simplistic since no family symmetry is present. One can circumvent this problem by introducing a family or horizontal symmetry, but more predictive results are obtained in the $SO(10)$ framework.

In $SO(10)$ all fermions of one family are placed in a **16** spinor supermultiplet and carry the same family charge assignment:

$$\mathbf{16}_i(u_\alpha,\ d_\alpha,\ u_\alpha^c,\ d_\alpha^c,\ \ell,\ \ell^c,\ \nu_\ell,\ \nu_\ell^c)_i, \quad i = 1, 2, 3. \tag{11}$$

Massive pairs of (**16**, $\overline{\mathbf{16}}$)'s and **10**'s may also be present. The Higgs Fields may contain one or more $\mathbf{45}_H$'s and pairs of $\mathbf{16}_H$, $\overline{\mathbf{16}}_H$ which break $SO(10)$ down to the SM, while $\mathbf{10_H}$ breaks the electroweak group at the electroweak scale. A $\overline{\mathbf{126}}_H$ or effective $\overline{\mathbf{16}}_H \cdot \overline{\mathbf{16}}_H$ field can generate superheavy right-handed Majorana neutrino masses.

With an appropriate family symmetry introduced, a number of texture zeros will appear in the mass matrices. These will enable one to make some well-defined predictions for the masses and mixings of the quarks and leptons, for typically fewer mass matrix parameters will be present than the 20 quark and lepton mass and mixing observables plus 3 right-handed Majorana masses.

Yukawa $t - b - \tau$ coupling unification is possible only for $\tan\beta = v_u/v_d \simeq 55$ in this minimal Higgs case described above. However, if a $\mathbf{16}'_H$, $\overline{\mathbf{16}}'_H$ pair is introduced with the former getting an electroweak-breaking VEV which helps contribute to H_d [17], or if the $\mathbf{16}_H$ of the first pair also gets an EW VEV, Yukawa coupling unification is possible for $\tan\beta \ll 55$. Such breaking VEV's can contribute asymmetrically to the down quark and charged lepton mass matrices. This makes it possible to understand large $\nu_\mu - \nu_\tau$ mixing, $U_{\mu 3} \simeq 0.707$, while $V_{cb} \simeq 0.040$. Moreover, the Georgi-Jarlskog mass relations [18],

$$m_s/m_b = m_\mu/3m_\tau \text{ and } m_d/m_b = 3m_e/m_\tau, \tag{12}$$

can be generated by the mass matrices with the help of the same asymmetrical contributions.

Models based on $SO(10)$ then differ due to their matter and Higgs contents as well as the horizontal family symmetry group chosen. Several selected illustrative examples of predictive $SO(10)$ GUT models are presented below, where some of their characteristic features are highlighted.

4.1 $SO(10)$ with $U(1)_H$

A model of this type has been presented by Babu, Pati, and Wilczek [19] based on dimension-5 effective operators involving

$$\text{Matter Fields}: \quad \mathbf{16}_1, \; \mathbf{16}_2, \; \mathbf{16}_3$$
$$\text{Higgs Fields}: \quad \mathbf{10}_H, \; \mathbf{16}_H, \; \overline{\mathbf{16}}_H, \; \mathbf{45}_H \tag{13}$$

The $\mathbf{16}_H$ develops both GUT and EW scale VEV's. With no CP violation, 11 matrix input parameters yield 18 + 3 masses and mixings. Maximal $\nu_\mu \leftrightarrow \nu_\tau$ mixing arises from the seesaw mechanism, while the SMA solar solution is preferred.

4.2 $SO(10)$ with $[U(1) \times Z_2 \times Z_2]_H$

Barr and Raby [20] have shown that a stable solution to the doublet-triplet splitting problem in $SO(10)$ can be obtained based on this global horizontal group. With an extension of the minimal Higgs content involved, Albright and Barr [21] have developed a model involving only renormalizable terms in the Yukawa superpotential with the following superfields present:

$$\text{Matter Fields}: \quad \mathbf{16}_1, \; \mathbf{16}_2, \; \mathbf{16}_3, \; 2(\mathbf{16}, \; \overline{\mathbf{16}})'s, \; 2(\mathbf{10})'s, \; 6(\mathbf{1})'s$$
$$\text{Higgs Fields}: \quad 4(\mathbf{10}_H)'s, \; 2(\mathbf{16}_H, \; \overline{\mathbf{16}}_H)'s, \; \mathbf{45}_H, \; 5(\mathbf{1}_H)'s \tag{14}$$

Ten matrix input parameters yield all 20 + 3 masses and mixings. A value of $\tan\beta \simeq 5$ is favored with $\sin 2\beta \sim 0.65$ obtained for the CKM unitarity triangle. Maximal $\nu_\mu \leftrightarrow \nu_\tau$ mixing arises from the lopsided texture of the charged lepton matrix. In this simplest scenario, the QVO solution is preferred, although by modifying the right-handed Majorana matrix the SMA or LMA solar solutions can be obtained with one or four more input parameters, respectively.

4.3 $SO(10)$ with $[SU(2) \times Z_2 \times Z_2 \times Z_2]_H$

Chen and Mahanthappa [22] have based a model on this family group with dim-5 effective operators involving the following superfields:

$$\text{Matter Fields}: \quad (\mathbf{16}, 2), \; (\mathbf{16}, 1)$$
$$\text{Higgs Fields}: \quad 5(\mathbf{10}, 1)_H's, \; 3(\overline{\mathbf{126}}, 1)_H's$$
$$\text{Flavon Fields}: \quad 3(\mathbf{1}, 2)_H's, \; 3(\mathbf{1}, 3)_H's \tag{15}$$

With no CP violation, eleven matrix input parameters yield $18 + 3$ masses and mixings, while $\tan\beta = 10$ is assumed. Maximal $\nu_\mu \leftrightarrow \nu_\tau$ mixing arises from the seesaw mechanism with symmetric mass matrices. The QVO solar solution is preferred, while the SMA solution or the LMA solution with $\tan^2\theta_{sol} < 1$ is difficult to obtain.

4.4 $SO(10)$ with $[U(2) \times U(1)^n]_H$

Blazek, Raby, and Tobe [23] have constructed such a model involving only renormalizable terms in the Yukawa superpotential with the following fields:

$$
\begin{aligned}
&\text{Matter Fields}: \quad (\mathbf{16}, 2), \ (\mathbf{16}, 1), \ (\mathbf{1}, 2), \ (\mathbf{1}, 1) \\
&\text{HiggsFields}: \qquad (\mathbf{10}, 1)_H, \ (\mathbf{45}, 1)_H \\
&\text{Flavon Fields}: \quad 2(\mathbf{1}, 2)_H's, \ (\mathbf{1}, 3)_H, \ 2(\mathbf{1}, 1)_H's
\end{aligned}
\tag{16}
$$

Sixteen matrix input parameters yield the $20 + 3$ masses and mixings with $\tan\beta \simeq 55$ required. CP violation occurs with $\sin 2\beta$ in the second quadrant for the CKM unitarity triangle. All solar neutrino solutions, SMA, LMA, LOW and QVO, are possible.

4.5 $SO(10)$ with $[SU(3) \times \text{unspecified discrete symmetries}]_H$

Berezhiani and Rossi [24] have proposed a model based on this group with the following structure:

$$
\begin{aligned}
&\text{Matter Fields}: \quad (\mathbf{16}, 3), \ (\mathbf{16}, \overline{3}), \ (\overline{\mathbf{16}}, 3), \ 2(\mathbf{16}, 3)'s, \\
&\qquad\qquad\qquad 2(\overline{\mathbf{16}}, \overline{3})'s, \ (\mathbf{1}, \overline{3}), \ (\mathbf{1}, 3), \ (\mathbf{10}, 3), \ (\mathbf{10}, \overline{3}) \\
&\text{Higgs Fields}: \quad (\mathbf{16}, 1)_H, \ (\overline{\mathbf{16}}, 1)_H, \ (\mathbf{54}, 1)_H, \ 2(\mathbf{45}, 1)_H's, \\
&\qquad\qquad\qquad 2(\mathbf{10}, 1)_H's \\
&\text{Flavon Fields}: \quad (\mathbf{1}, \overline{6})_H, \ 3(\mathbf{1}, 3)_H's, \ (\mathbf{1}, 8)_H
\end{aligned}
\tag{17}
$$

Fourteen matrix input parameters yield $18 + 3$ masses and mixings with moderate $\tan\beta$ assumed. Maximal $\nu_\mu \leftrightarrow \nu_\tau$ mixing arises from the lopsided texture of the charged lepton mass matrix. The SMA solar solution is preferred, while other solutions are possible with modification of the right-handed Majorana matrix.

5 Concluding Remarks

The most predictive models for the 12 "light" fermion masses and their 8 CKM and MNS mixing angles and phases are obtained in the framework of grand unified models with family

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symmetries. The $SO(10)$ models are more tightly constrained than $SU(5)$ models and are more economical than larger groups such as E_6 [25], where more fields must be made supermassive. In fact, some $SO(10)$ models do very well in predicting the $20 + 3$ "observables" with just 10 or more input parameters, depending on the model and type of solar neutrino mixing solution involved.

The SMA MSW solution is readily obtained in many unified models, since only one pair of states, ν_μ and ν_τ, are maximally mixed. Bimaximal mixing can be obtained in a smaller class of models, with the QVO solution having the more natural hierarchy with a pair of pseudo-Dirac neutrinos. The LMA MSW solution, if allowed, requires the most fine tuning, for two nearly maximal mixings must be obtained with a hierarchy of neutrino masses. For this latter solution, the right-handed Majorana neutrino masses typically span a range of $10^6 - 10^{14}$ GeV, while the lightest is typically 10^{10} GeV for the other solar neutrino solutions.

Unfortunately, while the experimental solar neutrino solution remains rather uncertain, unified model builders are not able to clarify the situation by predicting the outcome with any degree of certainty.

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